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10MCA12

First Semester MCA Degree Examination, June/July 2014
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If the set A has n elements then show that power set of A has 2^n elements. **(05 Marks)**

b. Using Venn diagram, prove that for any three sets A, B, C
$$A \Delta (B \Delta C) = (A \Delta B) \Delta C$$ **(05 Marks)**

c. If A, B, C are finite sets, prove that
$$|A - B - C| = |A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$$ **(05 Marks)**

d. A problem is given to four students A, B, C, D whose chances of solving it are ½, ⅓, ¼, ⅕ respectively. Find the probability that the problem is solved. **(05 Marks)**

2 a. Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology. **(05 Marks)**

b. Write Dual, Negation, Converse, Inverse, Contrapositive of the statement "If Kabir wears brown pant, then he will wear white shirt". **(05 Marks)**

c. Define $(p \uparrow q) \Leftrightarrow \neg (p \wedge q)$, represent $p \vee q$ and $p \to q$ using only $\uparrow$. **(05 Marks)**

d. Establish the validity of the statement:
If I like mathematics, then I will study.
Either I study or I fail.
Therefore if I fail, then I do not line mathematics. **(05 Marks)**

3 a. Verify the following is valid :
$\forall x\ [p(x) \vee q(x)]$; $\exists x \neg p(x)$
$\forall x\ [\neg q(x) \vee r(x)]$
$\forall x\ [s(x) \vee \neg r(x)]$ $\therefore \exists x \neg s(x)$ **(05 Marks)**

b. Prove that for all real no's x and y, if $x + y > 100$, then $x > 50$ or $y > 50$. **(05 Marks)**

c. Determine if the argument is valid or not.
All people concerned about the environment, recycle their plastic containers, B is not concerned about the environment therefore, B does not recycle his plastic containers. **(05 Marks)**

d. Negate and simplify: (i) $\forall x\ [p(x) \wedge \neg q(x)]$
(ii) $\forall x\ [p(x) \vee q(x) \to r(x)]$ **(05 Marks)**

4 a. Prove by mathematical induction that, for any positive integer n, 5 divided $(n^2 - n)$. **(06 Marks)**

b. For any real number x, define a sequence $\{c(n)\}$ by $c(1) = 1$ and $c(n) = 2 \times C\left(\left\lfloor \frac{n}{2} \right\rfloor\right)$ for $n \geq 2$, find c(67). **(07 Marks)**

c. For the Fibonacci sequence $F_0, F_1, F_2, \ldots\ldots\ldots$ prove that
$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$ **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

5 a. Define Cartesian product with example. For any non empty sets A, B & C prove that
$A \times (B - C) = (A \times B) - (A \times C)$ **(05 Marks)**

b. Let A and B be finite sets with $|B| = 3$. If there are 4096 relations from A to B then find $|A|$. **(05 Marks)**

c. Define Stirling numbers of second kind and if $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 5, 6\}$ then find
(i) How many functions are there from A to B?
(ii) How many are one to one functions?
(iii) How many are onto? **(05 Marks)**

d. Let ABC be an equilateral triangle of side 1 unit show that if we select 10 points in the interior, there must be atleast two points whose distance apart is less than 1/3. **(05 Marks)**

6 a. Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by ${}_aR_b$ iff "a is a multiple of b" represent
(i) The relation R as a matrix.
(ii) Draw its digraph.
(iii) List in-degree and out-degree of all vertices. **(07 Marks)**

b. Define composite relation with example. Let R be a relation from A to B and let S be a relation from B to C then prove that
(i) If $A_1 \subseteq A$ then $(S \circ R)(A_1) = A(R(A_1))$
(ii) $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$ **(07 Marks)**

c. Define poset with example. Let $A = \{1, 2, 3, 4, 6, 8, 12\}$. Define partial ordering relation R on A by ${}_xR_y$ iff x/y. Draw Hasse diagram. **(06 Marks)**

7 a. Let G be a set of all non zero real nos and let $a*b = ab/2$, show that (G, *) is an Abelian group. **(07 Marks)**

b. Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Verify that (A, A^2, A^3, A^4) form an Abelian group under matrix multiplication. **(07 Marks)**

c. State and prove the Lagrange's theorem. **(06 Marks)**

8 a. Construct a Decoding table (with syndromes) for the group code given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. Using this coding table decode the following received words:
1 1 1 1 0, 1 1 0 1 1, 1 0 0 0 0, 1 0 1 0 1 **(10 Marks)**

b. Determine whether $(Z, \oplus, \odot)$ is a ring with the binary operation $x \oplus y = x + y - 7$, $x \odot y = x + y - 3xy$ for all $x, y \in Z$. **(10 Marks)**

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